# Simplified Small-Signal Stability Analysis for Optimized Power System Architecture

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## I. Introduction

Power system architecture refers to the selection of system components and their connections in order to supply loads according to their requirements. The amount of possible architectural solutions for certain specifications can be excessive, thus the architectural selection has an important role in the overall optimization of a distributed power system. A methodology to design and optimize power distribution systems automatically is developed in [1], [2]. By utilizing behavioral modeling techniques [3], [4], fast simulation models are obtained, allowing the analysis of extremely large number of design options. This process results in optimized architectural solutions regarding the most fundamental system features. However, during the optimization process various features are neglected regarding the DC/DC converters: the solutions are obtained without considering the stability and dynamic behavior of the converters.

It is well known that the DC/DC converters are prone to impedance-based interactions introducing destabilizing effects to the system, due to their constant-power input-terminal behavior [5]. Therefore, it is necessary to include stability assessment as a part of the optimization methodology in order to assess the feasibility of the obtained solutions. For this purpose, the DC/DC converter is represented by a two-port network, composed of a set of measurable transfer functions known as G-parameters [6]–[9].

Traditional stability assessment method, based on minor-loop gain [10], is widely used in various interconnected systems covering different application areas [10]–[13]. This method utilizes the impedance-based minor-loop gain that is a ratio of the source or upstream subsystem output impedance and the load or downstream subsystem input impedance. Stability exists if the minor-loop gain satisfies the Nyquist criterion. Typically, the impedance-ratio-based stability region is presented as certain forbidden regions in the complex plane, out of which the minor-loop gain shall stay [14]–[16]. It is recently stated that the forbidden regions defined by the above mentioned methods occupy unnecessary large area in the complex plane, which can be reduced to a circle around the critical point (-1,0) without compromising the robustness of stability [17]. Similar forbidden region is earlier applied in [18] without giving justification for its usability. Passivity-based stability criterion (PBSC) is presented recently in [19], [20], where the passivity of a bus impedance is utilized to provide information regarding the stability thus avoiding the problem of analyzing the encirclement around the point (-1,0).

The purpose of this paper is to present a simplified method to systematically assess small-signal stability of a given power architecture as well as provide a measure of the robustness of system stability. The PBSC concept would provide a simple method to state the stability and, therefore, its applicability is evaluated. The concept of maximum peak criteria (MPC) derived from the behavior of the impedance-based sensitivity function [17] is proposed to be utilized to provide a measure of the system stability. Within control engineering it is a well known method to state robust stability of a closed-loop system [21]. The MPC concept provides the least conservative stability margins that can be expressed by a single number instead of providing both gain (GM) and phase (PM) margins.

The rest of the paper is organized as follows: Existing optimization methodology is briefly explained in Section II. Moreover, the applied modeling method suitable for commercial DC/DC converters is described. The applicability of the PBSC is evaluated in Section III and the proposed MPC-concept and

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its application is presented providing some practical examples. Section IV summarizes the proposed method in detail. The conclusions are finally drawn in Section V.

## II. Theoretical Background

The optimization of power architecture is a complex task and the amount of possible ways to connect various system components can be excessive. A methodology, based on simplified DC/DC converter models to obtain optimized power system solutions is briefly described. A modeling method that enables the system small-signal stability analysis is explained, showing how to obtain a minor-loop gain to analyze the influence of the source- or load-side impedance.

### A. Architecture Optimization

The power system design is typically desired to be optimized in terms of size, cost and efficiency. The main system component, DC/DC converter, is a major contributor on these features. Therefore, the optimization methodology is based on DC/DC converter models that only consider the converter size, cost and efficiency [3], [4]. These simple models enable fast analysis of various architectural solutions.

Based on the converter models, an architecture generation algorithm searches all suitable ways to connect these components according to the system specifications. The number of possible architectural solutions is huge and, therefore, the final optimization is performed utilizing evolutionary optimization techniques [1], [2]. Finally, the optimization process provides a selection of the most appropriate converters and a list of architectural solutions including the options with the smallest size, cost and losses as well as the solutions with the best trade-offs within these features. This methodology assists the design of distributed power systems as multiple architectural options can be assessed within a short time.

### B. Converter Modeling Method

The DC/DC converter models utilized for the optimization process consider in this case only the static properties of the converters [3], [4] in order to obtain fast simulation models for the analysis of large number of design options. Therefore, the optimized solutions are obtained without considering the stability and the dynamic performance of the converters. In order to analyze it, different modeling method that includes the effects of small-signal stability is needed. The model is required to be simple to implement as well as suitable for black-box modeling because the utilized converters are commercial with limited available information on their internal structure.

When the converters are interconnected to a system, adverse interactions might occur due to the converter sensitivity to the external impedances. This might lead to degraded converter transient performance or even instability. The interactions can be computed based on the internal dynamic representation that can be found by performing frequency response measurements through the converter input and output terminals. The input and output sources are assumed to be ideal. The corresponding four transfer functions describing the converter dynamic behavior according to Fig. 1 and (1) are:

- Audio susceptibility $G_{io} = \frac{\hat{u}_o}{\hat{u}_{in}}$
- Input Admittance $Y_{in} = \frac{\hat{i}_{in}}{\hat{u}_{in}}$
- Reverse Transfer function $T_{oi} = \frac{\hat{i}_{in}}{\hat{i}_o}$
- Output impedance $Z_o = \frac{\hat{u}_o}{\hat{i}_o}$

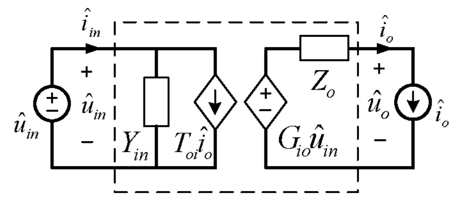


Fig. 1. Linear model of the converter with ideal source and load.

$$\begin{aligned} \hat{i}_{in} &= Y_{in}\hat{u}_{in} + T_{oi}\hat{i}_o \\ \hat{u}_o &= G_{io}\hat{u}_{in} - Z_o \cdot \hat{i}_o \end{aligned} \quad (1)$$

This modeling method enables the small-signal stability analysis of an interconnected system and its robustness. The influence of the source or load side impedance to the internal converter transfer functions in (1) can be analyzed as described in detail in [17]. The corresponding source and load-affected transfer functions can be given according to (2) and (3), respectively.

$$\begin{aligned} \hat{i}_{in} &= \frac{Y_{in}}{1+Z_sY_{in}} \cdot \hat{u}_{in} + \frac{T_{oi}}{1+Z_sY_{in}} \cdot \hat{i}_o \\ \hat{u}_o &= \frac{G_{io}}{1+Z_sY_{in}} \cdot \hat{u}_{in} - \frac{1+Z_sY_{in-sco}}{1+Z_sY_{in}} Z_o \cdot \hat{i}_o \\ Y_{in-sco} &= Y_{in} + \frac{G_{io}T_{oi}}{Z_o} \end{aligned} \quad (2)$$

$$\begin{aligned} \hat{i}_{in} &= \frac{1+Z_{o-oci}Y_L}{1+Z_oY_L} Y_{in} \cdot \hat{u}_{in} + \frac{T_{oi}}{1+Z_oY_L} \cdot \hat{i}_o \\ \hat{u}_o &= \frac{G_{io}}{1+Z_oY_L} \cdot \hat{u}_{in} - \frac{Z_o}{1+Z_oY_L} \cdot \hat{i}_o \\ Z_{o-oci} &= Z_o + \frac{G_{io}T_{oi}}{Y_{in}} \end{aligned} \quad (3)$$

According to (2) and (3), the source- and load-side minor-loop gains are $Z_sY_{in}$ and $Z_oY_L$, respectively. Based on these minor-loop gains, the small-signal stability and robustness can be analyzed subsequent to the system integration. Moreover, this modeling method enables more detailed assessment of the system-level interactions [9]. However, the focus of this paper is to obtain a systematic method for the stability and robustness analysis.

## III. Stability Assessment

In order to implement the stability assessment as a part of the optimization methodology, a systematic and straightforward analysis method is preferred. PBSC concept provides an alternative way to state the stability as a comparison to the traditional methods, eliminating the necessity of analyzing the encirclement of the point (-1,0). This method enables simple stability analysis and, therefore, its applicability is evaluated. For the optimized architecture, the obtained stability margins are desired to be the least conservative, i.e. optimized in terms of stability, guaranteeing the robustness. Middlebrook's criterion [10] is known to be too restrictive for general stability assessment. Therefore, applying a concept of forbidden regions, less conservative conditions for stability are obtained [14]–[16], where the Nyquist contour of the minor-loop gain is required to stay out of the predefined area and thus providing certain gain (GM) and phase (PM) margins for stability. A minimum forbidden region can be defined applying maximum peak criteria (MPC) [17]. This method is utilized to guarantee robust stability with the least conservative requirements.

### A. *Passivity-Based Stability Criterion (PBSC)*

Traditionally the stability analysis of distributed power systems is based on the impedance ratio known as minor-loop gain. Stability exists if the minor-loop gain satisfies the Nyquist stability criterion, i.e. does not encircle (-1,0). A newly proposed stability criterion [19] differs from the traditional methods: it imposes passivity of the overall bus impedance thus eliminating the necessity to analyze the encirclement of the point (-1,0). If passivity is satisfied for the total bus impedance, the system is stable. The conditions for passivity [19], [20] are:

- No RHP poles in $Z_{bus}$
- Re$\{Z_{bus}(j\omega)\} \geq 0$

Passivity provides a simple method to assess the stability: the Nyquist contour of the bus impedance is required to lie wholly on the right half plane (RHP) of the complex plane. Thus the real part of the bus impedance is required to be always positive, referring that the phase is always between $\pm 90\,^{\circ}$.

The PBSC provides only a sufficient but not necessary condition for stating the stability. A case, in which this concept discards a good solution, is illustrated by the following example. Fig. 2 shows a measured output impedance of a voltage-mode-controlled synchronous buck converter ($U_{in} = 12V, U_o = 5V, I_o = 2A, f_{sw} = 200kHz$) and a simulated input impedance ($R = -10\Omega, C_{in} = 17\mu$) of a cascaded DC/DC converter that operates as a constant power load, behaving like a negative resistor. The bus impedance for this system is defined as given in (4).

$$Z_{bus} = \frac{Z_{in}Z_o}{Z_{in}+Z_o} = \frac{Z_o}{1+\frac{Z_o}{Z_{in}}} \tag{4}$$

It can be observed from Fig. 2 that up to 3kHz, Middlebrook's stability criterion $Z_o \ll Z_{in}$ is satisfied with sufficient margin. Therefore, the bus impedance at low frequencies is equivalent to the output impedance of the buck converter. Based on the phase behavior of the measured output impedance, it can be observed that the phase exceeds $90\,^{\circ}$. The stability analysis for this cascaded connection is performed by analyzing the Nyquist plots of the bus impedance as well a traditional minor-loop gain as shown in Fig. 3.

According to the PBSC, the bus impedance is not passive because its Nyquist contour does not fully lie on the RHP. Therefore, the information provided by the passivity-based criterion regarding the system stability, is not clear. Whereas utilizing the traditional stability assessment methods [14]–[16], the system is stable with good margins. Consequently, for the stability analysis of a distributed power system consisting of commercial converters, (i.e. a system where no active damping can be applied) PBSC is a poor method to assess the stability.

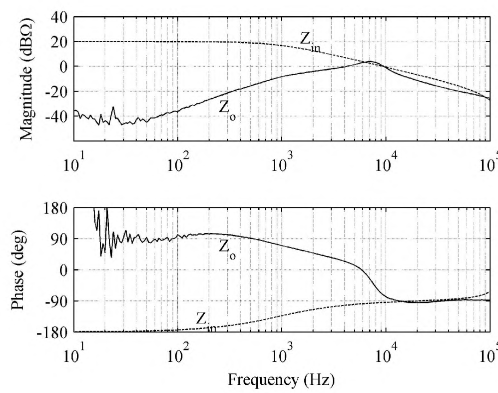


Fig. 2. Measured output impedance ($Z_o$) of a synchronous buck converter and simulated input impedance ($Z_{in}$) of a cascaded connected converter.

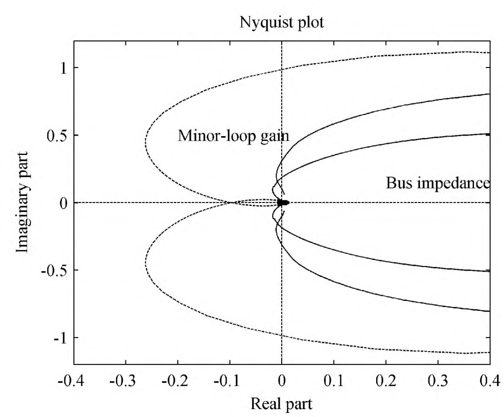


Fig. 3. Nyquist contour of the bus impedance and the minor-loop gain.

### B. *Maximum Peak Criteria*

A minimum forbidden region in the complex plane can be defined applying maximum peak criteria (MPC) [17]. This concept is well known in control engineering to define robust stability of a closed-loop system [21]. It is based on sensitivity function, defined in (5) where L denotes the system loop gain.

$$S = \frac{1}{1+L} \tag{5}$$

The critical area in the vicinity of the point (-1,0) determines the robustness of stability i.e. adequate gain (GM) and phase (PM) margins. Therefore, the measure of performance is assessed according to the closeness of the loop gain to the critical point (-1,0). This minimum distance is given as $1/M_s$ where the $M_s$ denotes the maximum value of the sensitivity function $\mid S_{max} \mid$. Low phase or gain margins of the loop gain L would cause resonant behavior (i.e., peaking) in the converter closed-loop transfer functions. The amount of this peaking is assessed based on the maximum peak of the sensitivity function and the corresponding margins for stability are given in (6) [21].

$$\begin{aligned} PM \geq & \; 2arcsin(\frac{1}{2 \mid S_{max} \mid}) \\ GM \geq & \; \frac{1}{1-\frac{1}{\mid S_{max} \mid}} \end{aligned} \tag{6}$$

For instance, a maximum peak of 5dB in the sensitivity function provides minimum margins of 7dB GM and 33$^\circ$ PM. The impedance-based minor-loop gain forms a similar sensitivity function (7) as the loop gain L in (5).

$$S = \frac{1}{1+ML} \tag{7}$$

The ML can be the source or the load side minor-loop gain. Therefore, low margins (GM and PM) in the minor-loop gain would cause peaking in the corresponding sensitivity function and consequently in the internal transfer functions of the converter (2), (3), degrading the transient behavior as demonstrated in [22].

Based on this concept, the MPC-based forbidden region is defined as a circle having its center at (-1,0) and the radius of $1/M_s$. In Fig. 4 the highlighted area illustrates the MPC-based forbidden area, having the maximum peak of 2 (6dB) compared to the regions in [14]–[16].

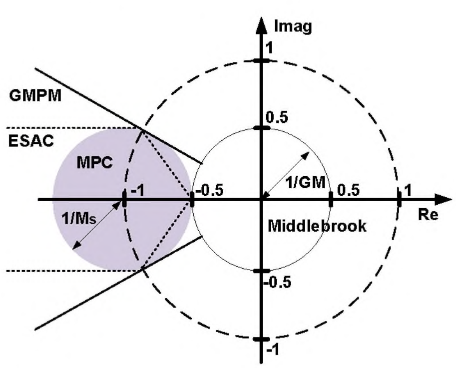


Fig. 4. The MPC-based forbidden region with the ESAC and GMPM regions.

This MPC-based forbidden region occupies the minimum area in the complex plane guaranteeing robust stability: the minor-loop gain is required to comply with the Nyquist criterion as well as to stay out of the circular forbidden area. The MPC-based forbidden region is determined by the maximum allowed peak of the sensitivity function and the area is, therefore, definable according to the robustness requirements of a particular application.

### *C. Application of the MPC-concept*

The robustness of the stability can be most reliably determined at the interface closest to the direct input or output of the converter power stage as explicitly demonstrated in [17]. In addition, the operation point where the frequency response measurements are performed influences on the stability margins. Few practical examples based on measurement data and simulations illustrate the application of this concept.

The peak sensitivity function provides information of the stability margins: the lower the peak value, the better in the sense of robust stability. The MPC- based forbidden region utilized in the following examples is obtained selecting the maximum peak 2 (6dB), corresponding to minimum margins of GM =6dB and PM $\approx 29^\circ$. Fig. 5 shows the Nyquist contour of two minor-loop gains. Minor-loop gain 1 corresponds to the same cascaded connection utilized in Section III A. The second plot presents the minor-loop gain formed between a measured input impedance of Ericsson Power Module, PMB 8518TP (12V, 3.3V, 10A) and an input filter that is designed to comply with the Middlebrook's criterion with a GM of 6dB.

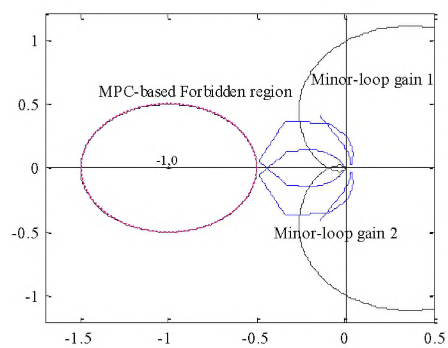


Fig. 5. Minor-loop gains 1(black line) and 2 (blue line), formed based on measured $Z_o$ of a buck and simulated $Z_{in}$ of a cascaded converter and measured $Z_{in}$ of the PMB converter and filter $Z_o$, respectively with the MPC-based forbidden region.

Based on visual observation, neither of the plots encircles the point (-1,0) nor intersects the forbidden region thus guaranteeing the minimum margins of 6dB GM and 29$^\circ$ PM. For systematic stability analysis, the same information is easily obtained computationally. The peaking of both minor-loop gains can be computed: 1.25 (1.92dB, minor-loop gain 1) and 1.9 (5.8dB, minor-loop gain 2) at the frequency of 3.46 kHz and 447 Hz, respectively. These values correspond to the minimum distance, $\frac{1}{M_s}$, between the Nyquist contour and the point (-1,0): 0.8 for minor-loop 1 and 0.52 for the second plot. The computed peaks of the sensitivity functions are lower than the predefined value for the MPC-based circle, guaranteeing the robustness.

The following example, described in detail in [17] demonstrates how the excessive peaking of the sensitivity function influences on the converter performance. The system consists of a bus converter, and two identical point-of-load converters, POL1 and POL2 as shown in Fig. 6 with the system specifications. The source-side minor-loop gain of the POL2 is measured at two operating conditions: firstly POL1 is operating at full load (4A) and POL2 at 1A and secondly POL2 is operating at full load (4A) and POL1 at 1A. These minor-loop gains are shown in Fig. 7 together with the MPC-based forbidden region.

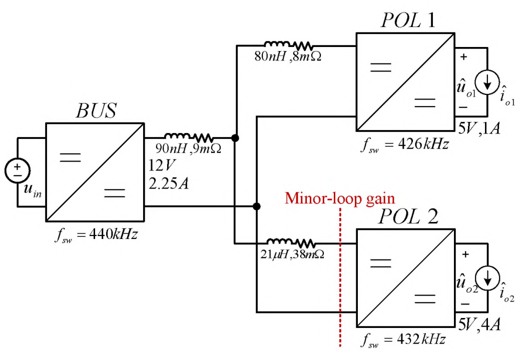


Fig. 6. Distributed power system consisting of a bus converter and two POL converters.

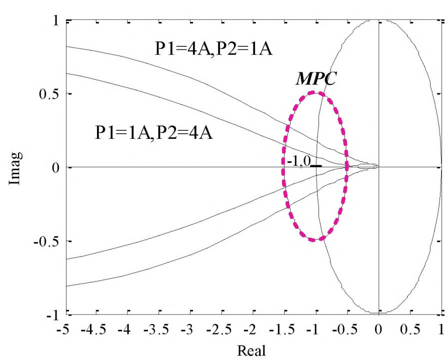


Fig. 7. The Nyquist plots of the same minor-loop gain at both operating conditions.

Both minor-loop gains intersect the MPC-based forbidden region. However, depending on the operational conditions, the stability margins vary. The peaking during the first operating condition is computed to be 13.7dB and in the second condition, 23.7dB. In both cases, the predefined peaking value 6dB is exceeded. When the POL2 is operating at full load the worst case stability margins are 0.6dB of GM and 4° of PM, respectively. The influence of this peaking can be observed from the measured output impedance of the POL2 in both operating conditions as shown in Fig. 8.

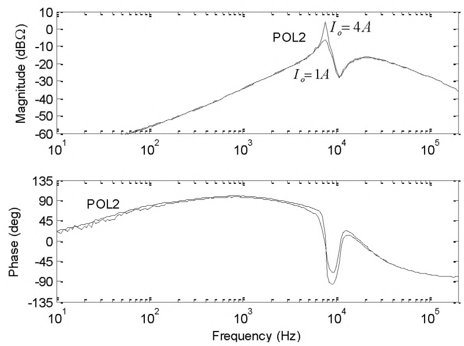


Fig. 8. Measured source-affected output impedance of the POL2 at both operating conditions.

The converter output impedance is affected due to the source impedance (system interconnection) implying that the performance is deteriorated. This can be observed in time-domain, when a load step is applied at the output of POL2 as shown in Fig. 9. The damped oscillation in the output voltage response is due to the resonance in the source-affected output impedance of POL2.

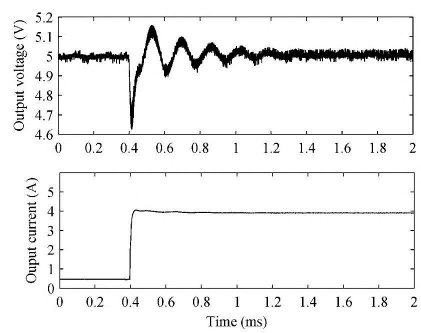


Fig. 9. Measured time-domain behavior of POL2 during a load step from 0.5 to 4A (250mA/$\mu$s) at the output of POL2.

## IV. Proposed Methodology

The optimization process provides a set of architectural solutions that are optimized regarding the system size, cost and efficiency. In order to obtain information of the system stability and robustness, the presented concepts are applied. The system stability is analyzed based on the encircling of the minor-loop gain around the point (-1,0). By utilizing the MPC-concept, one single value that combines the effects of both margins, is obtained thus enabling systematic stability assessment. In addition, as the architectures are desired to be comparable, a single number to provide a measure of the whole system stability is required.

### A. Implementation

Each converter within the power architecture is represented as its two-port model that contains the information from the internal converter dynamics. This modeling structure allows the interconnection of the DC/DC converters to form a system according to the given architectural structure. The stability analysis is based on the minor-loop gain and divided into two parts:

- System stability analysis according to the Nyquist criterion
- Robustness analysis based on the MPC-concept

Unstable system is detected by assessing whether the Nyquist contour of the minor-loop gain encircles the point (-1,0) or not. For a stable system, the stability margins are assessed. In order to correctly predict the robustness within the system, the source and load side minor-loop gains are analyzed for each converter. The robustness of the stability is stated by computing the maximum value of the minor-loop gain based sensitivity function in (7). This number is utilized to evaluate whether the system interconnection might deteriorate the converter performance.

### B. Performance metrics

The concept of maximum peak sensitivity function provides the least conservative performance metrics for system stability. One of the objectives is to compare architectural solutions in terms of stability and, therefore, it is important to obtain a single number that provides an overall measure of the whole system stability. Thus the robustness information (computed maximum peaks) of each minor-loop gain can be combined by applying different norms such as:

- Average
- Euclidean
- Infinite norm (worst case)

For this purpose, the power architecture can be represented by a tree structure [1], [2] as shown in Fig. 10 and utilize an algorithm to analyze each branch. Every converter connection is presented by a single number and compared to a predefined value of the maximum peak that is set based on the MPC forbidden region, according to the system specifications. In case any of the computed values is larger than the predefined maximum peak, a warning is provided to inform about the deteriorated stability margins. Otherwise, the algorithm analyses the source- and-load side minor-loop-gain-based sensitivity functions for each converter performing an operation according to the selected norm. For example, the infinite norm captures the largest value, i.e. the worst case, thus detecting the weakest point of the system in terms of robustness and utilizes it as a figure of merit. Finally, a single number states the robustness of the whole system enabling the comparison of different architectural solutions in terms of stability. The objective for future work is to analyze the most appropriate norm to compare the architectural solutions.

## V. Conclusion

This paper presented a simplified small-signal stability analysis method for a given power system. It was shown that the passivity-based criterion is not applicable for the stability assessment of a system consisting of commercial converters. The applied MPC-concept, based on the minor-loop-gain- based sensitivity function provides the least conservative method to obtain stability margins for optimized power architecture. In addition, the advantage of this criterion is that it considers the combined effects of both margins thus providing a single number for stating the robustness of stability. This is a systematic analysis approach enabling the integration of the proposed method into the optimization methodology.

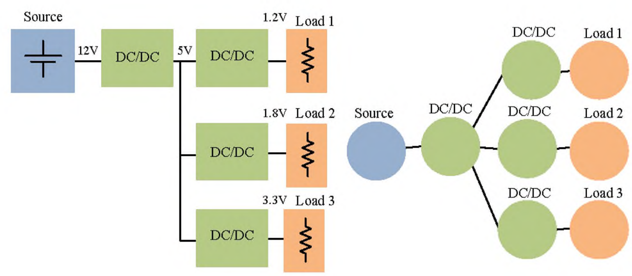


Fig. 10. Power architecture and corresponding tree graph.